Government Education Funding and Government Higher Education Investment in Uganda

*JIMMY ALANI[1], PATIENCE ATUHAIRWE[2], DAVID L. OKODEL[3]
[1,2,3]*Department of Economics and Business Education,*
Faculty of Education and Humanities, Gulu University, Gulu, Uganda
**Corresponding author: alanijimmy@gmail.com*

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Abstract

The paper examines the relationships between government funding of education and government investment in higher education in Uganda during the 2005 to 2020 period. The major hypotheses of the study are that government funding of higher education (GHE) depends on government investment in education (GIE) and the reverse is true. During the given period, government spending on education as a percentage of GDP for Uganda was on average 2%, while the world average was 4% (World Bank, 2022). This implies that low levels of government funding are responsible for the low levels of government investment in higher education in Uganda. Therefore, the study aims to estimate the government investment function of higher education and the production function of government funding in Uganda during the given period. The analyses of the relationships between government education funding (GIE) and government investment in higher education (GHE) are conducted by using the generalised least squares (GLS) method. The dataset for the study was obtained from Uganda's Ministry of Education and Sports (MoES), Education Sector Plan. The effects of 1% government spending on primary education, secondary education, technical and vocational education training (GTVE) and GHE on GIE were as follows: 0.354%, 0.086%, 0.044% and 0.063% per annum, respectively. This implies that in Uganda, GHE contributes very little (0.063%) to GHF compared to the contribution of GPE to GIE (0.35%). Meanwhile, a 1% increase in GIE might have caused growth in GHE and GTVE to rise by 11.45% and 17.18%, respectively. Hence, the paper suggests increasing GIE, GTVE and GHE adequately.

Keywords: *Government funding; Higher education investment.*

Introduction

Globally, knowledge is the main source of long-term economic growth for all the economies. Development in the modern market economies show that education and investment in education are some of the most important priorities of national strategy, national policy, economic and technological progress (Ng, 2003; Psacharopoulos & Woodhall, 1985; Slaus & Jacobs, 2011;Zoran Tomic, 2017). Meanwhile, investment trends in human capital and knowledge generate the revision of economic theory and models (Bassanini et al., 2005; Jones & Romer, 2009). The traditional "production function" concentrates on labour, material, capital and energy as the main factors. Moreover, knowledge and technology are considered as external factors that affect production. New analytical approaches have been developed to enable the inclusion of knowledge directly in the production function (Hanushek & Woessmann, 2015; Mohamed et al., 2021; Phale et al., 2021; Taupo, 2008; Zoran Tomic, 2017).

Investment in knowledge has the potential of increasing the production capacity more than any other factor of production and to transform from less competitive products to more competitive new products and processes. Such investments in knowledge increase the rate of return on investment (Zoran Tomic, 2017). They are key to long-term economic growth (Zoran Tomic, 2017). Human capital theory postulates a positive relationship between the levels of education, the main way of acquiring human capital, and labour productivity (Bassanini et al., 2005; Mohamed et al., 2021; Nowak & Dahal, 2016; Phale et al., 2021).

Thus, higher levels of education, ceteris paribus, contribute more to economic growth than lower levels of schooling. In a knowledge-based economy, human capital is a major building block for the sustainable growth path (Harmon et al., 2003; Nedic et al., 2020; Nowak & Dahal, 2016; Sabur et al., 2021).

Meanwhile, basic education provides the foundation for learning, but tertiary education develops core skills that promote creative and critical thinking. Therefore, tertiary education is necessary for any nation to develop, advance and maintain sustainable growth. A nation's effective returns on education contribute towards its development and advancement, and yield benefits from generation to generation. In education, teachers provide positive guidance to the future generation and help to accelerate the development of the nation (Islam et al., 2016; Nowak & Dahal, 2016).

The objectives of the paper are as follows: (a) to estimate the long-run causal relationships between government expenditure on education and government higher education investment in Uganda from 2005 to 2020, by using the generalised least squares method; (b) to systematically distil, from the Cobb-Douglas production function, the linear relationship between components of government spending on education and its investment in education itself; and (c) to theoretically and empirically test the postulate that returns on either primary or secondary education have been greater than those on higher education.

Our first objective is mainly to estimate the effect of higher education on economic growth in Uganda. Therefore, we systematically estimate the effect of higher education investment on government spending in Uganda during the given period. An estimate of the relevant parameters involves using the Cobb-Douglas production function both in theoretical and empirical analyses. On the other hand, we employ Wagner's law in both theoretical and empirical analyses. In the second objective, the paper deals with components of government spending on education expressed in terms of primary, secondary, technical and vocational training, tertiary (higher) education, and administration. The third objective postulates that returns on government spending on higher education has been greater than those on government spending on primary or secondary education

The justification of the study is that few research works have ever empirically examined the linear relationship between government spending on education and government investment in higher education, as observed by Jeyhoon Tabar et al. (2017) and Ranjan and Chintu (2013). Meanwhile, Wagner's law is the one of the theoretical discussions that present a relationship between the size of the public sector and economic growth. The law postulates that economic growth results in government expenditure growth. Wagner is considered to be the first researcher to discover the positive relationship between the level of economic development and the size of the public sector (Jeyhoon Tabar et al., 2017; Ranjan & Chintu, 2013). However, in our paper, we postulate that government expenditure growth leads to an increase in economic growth and vice versa to maintain a full employment equilibrium in the economy. We derive models for empirical tests by using the Cobb-Douglas production function in terms of making (a) government funding of education a function of income; (b) government spending on education a function of government financing of higher education; and (c) income a function of government investment in higher education.

The paper focuses on examining the relationship between government spending on higher education and its investment in higher education. However, previous literature on education expenditure focuses mainly on the effects of government investment in education on economic growth, for instance Augustine (2020), Bah and Kpognon (2021), Dini and Aji (2022), Nyoman et al. (2021) and Sergeeva et al. (2022). The results of previous research studies regarding Wagnar's law are mixed, and they arrive at different conclusions depending on the selected countries, periods and model specifications. The first group of researchers, for instance Magdalena and Suhatman, (2020), Maneejuk and Yamaka (2021), Trabelsi (2017) and Zoran Tomic (2017) found positive relationships between government education expenditures and economic growth. In contrast, few research studies in the second and third categories found negative or no relationship between the two variables.

Review of Literature

Theoretical of review Wagner's law and Keynesian hypothesis

Wagner's law

Wagner's (1835–1917) law (hypothesis) sates that there is a functional cause and effect relationship between the economic growth of an industrialising economy and the relative growth of its government spending (Moheeth, 2017). Wagner postulates that there are inherent tendencies for the activities of different sectors of a government (such as education and health) to increase both intensively and extensively. For instance, government spending on education and health sectors constantly generates new functions. Meanwhile, government performs both old and new functions fully and more efficiently (Moheeth, 2017). Wagner's law of increasing government spending has become a universal truth in recent years. As such, the economic growth of every country has always been accompanied by increasing government activities and hence increasing government expenditure (Moheeth, 2017).

Empirical evidence supports the hypothesis of a continuous upward trend in government activities (Moheeth, 2017). Moheeth supports the argument that government investment is justified because investment required in some areas is so high (for example railways) that the private sector is unable to finance it (Barro, 1988; Grier, 1989). Wagner reasons that economic growth is the cause of the growth of the public sector. Economic growth increases the per capita income of countries in the process of industrialisation. As a result, the share of public expenditure in the total expenditure increases over time (Jeyhoon Tabar et al., 2017).

Several scholars have examined the relationship between economic growth and government spending and arrived at contradictory conclusions (Chu et al., 2020; Magdalena & Suhatman, 2020; Maneejuk & Yamaka, 2021; Padhi, 2016). According to Nyasha and Odhiambo (2019), these empirical research findings fall into three categories. The first group found a positive relationship between economic growth and government expenditure (Ali et al., 2013; Kimaro et al., 2017; Nyasha & Odhiambo, 2019; Raghupathi & Raghupathi, 2020). Meanwhile, the second group empirically found a negative relationship between economic growth and government expenditure (Altunc & Aydın, 2013; Lupu et al., 2018). Whereas, the third group of researchers did not find any significant relationship between economic growth and government spending (Mokoena et al., 2020). Economic theory postulates that public spending is the driving force behind economic development (Nyasha & Odhiambo, 2019; Dudzevičiūtė, 2023).

In scientific literature, there are three dominant approaches to the relationship between economic growth and government spending. One, the Keynesians believe that an increase in government spending drives economic growth through the multiplier effects (Iniguez-Montiel, 2010). Two, in contrast, the Wagner group contends that increases in government spending are caused by economic growth (Wagner, 1892). According to the Wagner followers, there are three main sectors that influence the growth of public expenditure. They are administrative costs, social protection and welfare provision (Laboure & Taugourdeau, 2018). Three, the classicals believe that increases in government spending have a negative impact on private investment and hamper economic development in the long run (Chen et al., 2022; Palley, 2013). Proper understanding of how to allocate financial resources to different activities is important to both policymakers and economic researchers. That is why the 20th century was dominated by increasing levels of government spending and activities (Tâm et al., 2016) arising from past economic and financial crises, or increasing demand for social services (Dudzevičiūtė, 2023; Kutasi & Marton, 2020).

Keynesian hypothesis

Keynes states that *"public expenditure increases have positive effect on the economic growth"*(Keynes, 1937). In addition, Keynes argued that public expenditure is an exogenous factor. The Keynesian economic theory postulates that government spending can stimulate economic growth, particularly in times of economic downturn. Government has the power to stimulate aggregate demand by increasing its expenditures, and consequently leading to economic growth (Akmal & Fayzullokh, 2023; Babatunde, 2018). This implies that the direction of causality in the relationship between public expenditures and national income is

from government sector expenditures to income (Babatunde, 2011). Keynesians believe that public sector expenditures are real means of boosting and increasing economic activities. As a result, public sector expenditures stabilise short-term fluctuations in the total expenditures. Literature on endogenous growth shows that government expenditures directly affect the production functions of the private sector (Jeyhoon Tabar et al., 2017).

Health and education are two sectors that are very important for an economy. UNESCO (2020) emphasises that health and wellbeing are built on a foundation of high-quality education. As a result, people need to know how to prevent illness and disease if they want to live long and healthy lives. Consequently, children and adolescents must be properly nourished and their good health maintained in order for them to learn more effectively. Meanwhile, both health and education are fundamental, universal human rights required for both social and economic advancement. Government spending is a powerful fiscal policy tool that can be used to control the entire economy, including health, education in general and higher education in particular (Atabukum et al., 2020).

Keynesian theory involves fiscal arrangements of public disbursements to stimulate economic growth. Keynesians believe that public spending can be used to affect economic growth positively. That is because increasing government expenditures leads to expansion in the rate of employment, cost-effectiveness, venture capital multiplier effects and increased demand. As a result, government spending supplements the aggregate demand, which enhances an increased output depending on expenditure multipliers (Egbo et al., 2016).

In contrast, according to Wagner, economic growth is a natural cause of public sector expenditures (Atabukum et al., 2020). More recent endogenous growth theories hold a strong view, indicating that the effect of government expenditure on economic growth depends mainly on the efficiency of public spending and the sectors it is based on, particularly on investments in human capital and infrastructure (Akmal & Fayzullokh, 2023; Arauco et al., 2022). The relationship between government expenditures and economic growth still remains a complex and multifaceted issue. This relationship appears to be affected by several factors, including the type and efficiency of government expenditure, the level of economic development and the specific economic context (like levels of education). Therefore, further research is needed to provide more accurate results for policymakers to enable them to optimise budget expenditures to promote economic growth (Akmal & Fayzullokh, 2023).

Empirical review of literature

Empirics of Wagner's law

Qi (2016) examined the effects of government education expenditure on economic growth in China by considering the spatial third-party spillover effects. He found that aggregate government education expenditure in China has a significant positive effect on economic growth. However, expenditure in different education sectors shows different results. Government education expenditure below higher education is positively related to local economic growth. Meanwhile, the effect of education expenditure on higher education is insignificant.

Empirical studies in public educational spending and economic growth have yielded inconsistent and inconclusive results. Mercan and Sezer (2014) found that public expenditure on education had a positive effect on Turkey's economic growth from 1970 to 2012. Similarly, Al-Yousif (2008) uses time-series data covering the period 1977–2004, in six countries (Saudi Arabia, Kuwait, Oman, UAE, Bahrain and Qatar) and shows a positive effect of education expenses on economic growth. Meanwhile, Haini (2020) contends that government expenditure on education across provinces in China accelerated economic growth from 1996 until 2015. Also, a meta-analysis study on the effects of government education and health expenditures on economic growth reveals that public education expenses positively affect economic growth (Awaworyi Churchill et al., 2015; Egbo et al., 2016; Suwandaru et al., 2021).

To justify Wagnar's law, Fabrizio (2016) carried out a meta-analysis proof on 29 empirical studies. The meta study revealed that out of 29 reviews, only 14 depicted positive effects of public educational spending on welfare, 12 exhibited negative effects and three showed no significant effects of public spending on welfare. Meanwhile, in Africa, S.A. Babatunde (2018) examined the effect of government education

expenditure on economic growth in Mozambique using a co-integration approach and quarterly data between 1996 and 2012. By using co-integration and error-correction analysis they found that a long run relationship existed between economic growth and government expenditure in Mozambique (Atabukum et al., 2020).

Touruam et al. (2014) investigated effects of government spending in tertiary education on per capita income in Nigeria from 1990 to 2011. Their findings indicated that government spending on education had significant positive effects on economic growth. By using a 2SLS approach, Urhie (2014) examined whether in Nigeria the effects of public educational expenditures on growth depended on its composition. His results revealed that public educational expenditures had positive direct effects on economic growth. A 1% increase in education caused a 4% increase in economic growth, ceteris paribus. However, when he disaggregated educational expenditures into capital and recurrent expenditures, the effects became different. A 10% increase in the proportion of total public expenditure allocated to recurrent expenditure on education caused economic growth to increase by 1.7%, ceteris paribus (Atabukum et al., 2020).

Empirical literature indicates that Ukwueze (2014) and Okoro (2013) invalidate Wagner's law. Meanwhile, Anoke et al. (2016) and Dada and Adewale (2013) found bidirectional causality that supports Wagner's law. However, this study used the Cobb Douglas production function disaggregated components of government spending on education and examined the causal relationship among economic growth variables and economic growth.

Wagner's law and Keynesian hypothesis dynamics

Wagner's law, also known as the "law of increasing state activity" (Arestis et al., 2021), postulates that causality runs from economic growth to government spending (Paparas et al., 2019). However, all over the world, economists hold conflicting views on Wagner's law (hypothesis) that an increase in economic growth causes government expenditure. Empirical studies indicate that Wagner's law has been experienced in various countries (Paparas et al., 2019). Meanwhile, empirical studies show that the Keynesian hypothesis also exists in many countries (Babatunde, 2018). Keynesians argue that aggregate demand has the positive effect of autonomous government expenditure on economic growth (Arestis et al., 2021). Government expenditure can increase national income through a multiplier effect on aggregate demand, leading to economic growth. They believe that government expenditure is the engine that drives economic growth. Government participation in economic activities in modern times is required because government can correct short-term distortions in an economy (Aluthge et al., 2021).

Government also provides basic services such as health, education, communication and transportation. Consequently, government expenditures affect the wellbeing of citizens and business environment of the private sector (Abu-Eideh, 2015; Aluthge et al., 2021). Keynesian theory (Keynes, 1936) considers government spending as an exogenous (fiscal) policy variable that can be used to influence growth and development in the short run. Whereas Wagner's law (Wagner, 1890) considers economic growth as the cause of government spending in terms of its investment in education as a whole, and its spending on education at primary, secondary, technical and vocational, higher and administrative levels (Tang & Lai, 2022).

Keynesian economists prefer to use public expenditure in promoting growth and development by stimulating aggregate demand, while Wagner does not provide any mathematical formulation in order to examine his law. Our contribution is to give theoretical and empirical answers to one critical question that asks whether Wagner's law is true. Wagner's law states that "public expenditure rises faster than national output" (Gatsi et al., 2019). To provide meaningful answers, we consider three variables: output (GDP) denoted by (Y), government spending denoted by (G) and output excluding government spending denoted as (Q), where Q=Y-G=Q[Y(G)].

We then reason that both the Cobb-Douglas and the linear production functions are correct models that yield the same values of required parameters. Therefore, the two models can be represented (a) as given in equation (2.1) for the Cobb-Douglas production function and (b) as given in equation (2.2) for the linear production function.

$$Y = G^{\alpha} Q^{\beta}. \qquad (2.1)$$

$$Y = \alpha G + \beta Q \qquad (2.2)$$

where and are the parameters to be determined by regression. Partial differentiation of both equations (2.1) and (2.2) with respect to provides equations (2.3) and (2.4) respectively.

$$\frac{\partial Y}{\partial G} = \alpha \frac{Y}{G} \qquad (2.3)$$

$$\frac{\partial Y}{\partial G} = \alpha \qquad (2.4)$$

Hence, in equations (2.4) and (2.3) the marginal product of is because the regression of on and in equation (2.4) will always make each of the parameters to be 1. This implies that equation (2.3) will always provide correct solutions pertaining to the long-run production function by rewriting equation (2.3) and representing it as given in equation (2.5). (Also see Appendix 2.)

$$1 = \left(\frac{1}{\alpha}\right)\frac{G}{Y}, \qquad \alpha < 1 \qquad (2.5)$$

which shows that the government sector will not always increase at a rate higher than that of output.

In contrast, according to our law, the transformation of equation (2.1) into a logarithm and rearranging the terms indicate that "economic growth would often increase at a rate higher than that of government sector", suggesting that Wagner's law may be untrue.

$$\log G = \frac{1}{\alpha}\ log Y - \beta log Q \qquad (2.6)$$

However, both Wagner's law and our hypothesis are correct due to the existence of an equilibrium, in that economic growth affects government sector with the same magnitude at which the government sector influences economic growth. (See Appendix (2A) for details.)

Methodology

This present section involves building the models that were employed in the analysis of the relationship between government education funding and higher education investment along with government spending on primary, secondary, business, technical and vocation education training and education administration. The section consists of three subsections: (a) Government spending in each education sector affects its spending on education; (b) Deriving functions of government investment in each education sector; and (c) Deriving Wagner' law from the Cobb-Douglas production function.

Government spending in each education sector affects its spending on education

The government education production function that is composed of expenditure on primary education and other expenditures that are not spent on primary education can be represented as follows:

$$Y_{gt} - X_{pt}^{a_p}\ X_{pnt}^{a_{pn}}\ e^{u_t} \qquad (3.1)$$

where in year (Y_{gt}) is government total expenditure on education, (X_{pt}) is government expenditure on primary education, (X_{pnt}) is other government expenditures not spent on primary education, u_t is the error term and α_p, α_{pn} are coefficients of (a) primary education expenditures and (b) government expenditure on education excluding government expenditures on primary education respectively.

Here, government expenditure on primary education and other government expenditures not spent on primary education are treated as inputs and total government expenditure on education is viewed as output within a given year. Therefore, the marginal physical product of government spending on primary education is given by

$$\frac{\partial Y_{gt}}{\partial X_{pt}} = \alpha_{pt}\ \frac{Y_{gt}}{X_{pt}} \qquad (3.2)$$

This implies that within any given year, a 1% increase in the growth of government expenditure on primary education ($\partial X_{pt} / X_{pt}$) has the potential of causing growth in government expenditure in education ($\partial Y_{gt} / Y_{gt}$) to increase by α_{pt} per cent.

Similarly, the marginal physical product of government spending on education but excluding government spending on primary education can be represented as follows:

$$\frac{\partial Y_{gt}}{\partial X_{pnt}} = \alpha_{pnt} \frac{Y_{gt}}{X_{pnt}} \tag{3.3}$$

This implies that within any given year, a 1% increase in the growth of government expenditure on education but excluding government spending on primary education has the potential of causing growth in government expenditure on education to increase by per cent, other things remaining constant.

Meanwhile, total differentiation of equation (3.1) provides:

$$dY_{gt} = \frac{\partial Y_{gt}}{\partial X_{pt}} dX_{pt} + \frac{\partial Y_{gt}}{\partial X_{pnt}} dX_{pnt} \tag{3.4}$$

Manipulation of equation (3.4) provides the following:

$$1 = \frac{\partial Y_{gt}}{\partial X_{pt}} \frac{dX_{pt}}{dY_{gt}} + \frac{\partial Y_{gt}}{\partial X_{pnt}} \frac{dX_{pnt}}{dY_{gt}} \tag{3.5}$$

Substitution of equations (3.2) and (3.3) in equation (3.5) yields:

$$1 = \alpha_{pt} \frac{Y_{gt}}{X_{pt}} \frac{dX_{pt}}{dY_{gt}} + \alpha_{pnt} \frac{Y_{gt}}{X_{pnt}} \frac{dX_{pnt}}{dY_{gt}} \tag{3.6}$$

Hence, the effects of government spending on primary education and government spending in education excluding government spending in primary education on government spending on education can be represented more compactly and accurately as follows:

$$1 = \alpha_{pt} \frac{Y_{gt}}{X_{pt}} + \alpha_{pnt} \frac{Y_{gt}}{X_{pnt}} \tag{3.7}$$

We transform equation (3.7) appropriately to equation (3.8) in order to reflect the philosophical principle of causality (Mishkin, 2004, p.116; Rubi, 2004; Alani, Yawe & Mutenyo, 2022) involving government spending on education as a function of (a) government spending on primary education and (b) government spending on education excluding primary education, as follows:

$$1 = \alpha_{pt} \frac{Y_{gt}}{X_{pt-1}} + \alpha_{pnt} \frac{Y_{gt}}{X_{pnt-1}} \tag{3.8}$$

Similarly, the effects of government spending on secondary education and government spending in education excluding government spending in secondary education, on government spending in education can be represented more compactly and accurately as follows:

$$1 = \alpha_{st} \frac{Y_{gt}}{X_{st-1}} + \alpha_{snt} \frac{Y_{gt}}{X_{snt-1}} \tag{3.9}$$

Likewise, the effects of government spending on vocational education and government spending in education excluding government spending in vocational education, on government spending in education can be represented more compactly and accurately as follows:

$$1 = \alpha_{vt} \frac{Y_{gt}}{X_{vt-1}} + \alpha_{vnt} \frac{Y_{gt}}{X_{vnt-1}} \tag{3.10}$$

Likewise, the effects of government spending on higher education and government spending in education excluding government spending in higher education, on government spending in education can be represented more compactly and accurately as follows:

$$1 = \alpha_{ht} \frac{Y_{gt}}{X_{ht-1}} + \alpha_{hnt} \frac{Y_{gt}}{X_{hnt-1}} \tag{3.11}$$

Similarly, the effects of government spending on administering education and government spending in education excluding government spending in administering education, on government spending in education can be represented more compactly and accurately as follows:

$$1 = \alpha_{qt} \frac{Y_{gt}}{X_{qt-1}} + \alpha_{qnt} \frac{Y_{gt}}{X_{qnt-1}} \quad (3.12)$$

Deriving functions of government investment in each education sector

From Equation (3.1) we can derive government investment in each education sector as a function of government investment in education. Thus, government investment in the primary education sector can be represented as a function of government funding of education as follows:

$$X_{pt} - Y_{gt}^{(1/a_p)} X_{pnt}^{-(a_{pn}/a_p)} e^{u_t} \quad (3.13)$$

Meanwhile, government investment in the secondary education sector can be represented as a function of government investment in education as follows:

$$X_{st} - Y_{gt}^{(1/\alpha_s)} X_{snt}^{-(\alpha_{sn}/\alpha_s)} e^{u_t} \quad (3.14)$$

Similarly, government investment in the vocational education sector can be represented as a function of government investment in education as follows:

$$X_{vt} - Y_{gt}^{(1/\alpha_v)} X_{vnt}^{-(\alpha_{vn}/\alpha_v)} \quad (3.15)$$

Likewise, government investment in the higher education sector can be represented as a function of government investment in education as follows:

$$X_{ht} - Y_{ht}^{(1/h_v)} X_{hnt}^{-(\alpha_{hn}/h_v)} e^{u_t} \quad (3.16)$$

Similarly, government investment in the administration of the education sector can be represented as a function of government investment in education as follows:

$$X_{qt} - Y_{gt}^{(1/\alpha_a)} X_{qnt}^{-(\alpha_{an}/\alpha_a)} e^{u_t} \quad (3.17)$$

In equations (3.13) to (3.17) we treat X_{pt}, X_{st}, X_{vt}, X_{ht}, X_{at} as outputs arising from the utilisation of X_{gt}, X_{pnt}, X_{snt}, X_{vt}, X_{ht}, X_{at}. Our theoretical framework is neoclassical in nature because it makes use of the Cobb-Douglass production function models. Therefore, the transformation of equation (3.17) into logarithm form provides equation (3.18). We use the philosophical causality principle (Rubi, 2009) involving government spending on primary education as a function of (a) government spending on education and (b) government spending on education excluding primary education to obtain

$$\log (X_{pt}) = (1/\alpha_p)\log (Y_{gt-1}) - (\alpha_{pn}/\alpha_p)\log (X_{pnt-1}) \quad (3.18)$$

Meanwhile, the influence of government spending on secondary education as a function of (a) government spending on education and (b) government spending on education excluding secondary education can be represented as follows:

$$\log (X_{st}) = (1/\alpha_s)\log (Y_{gt-1}) - (\alpha_{sn}/\alpha_s)\log (X_{snt-1}) \quad (3.19)$$

Similarly, the influence of government spending on vocational education as a function of (a) government spending on education and (b) government spending on education excluding vocational education can be represented as follows:

$$\log (X_{vt}) = (1/\alpha_v)\log (Y_{gt-1}) - (\alpha_{vn}/\alpha_v)\log (X_{vnt-1}) \quad (3.20)$$

Likewise, the influence of government spending on higher education as a function of (a) government spending on education and (b) government spending on education excluding higher education can be represented as follows:

$$\log (X_{ht}) = (1/\alpha_h)\log (Y_{gt-1}) - (\alpha_{hn}/\alpha_h)\log (X_{hnt-1}) \quad (3.21)$$

Similarly, the influence of government spending on administering education as a function of (a) government spending on education and (b) government spending on education excluding administration of education can be represented as follows:

$$\log (X_{at}) = (1/\alpha_a)\log (Y_{gt-1}) - (\alpha_{an}/\alpha_q)\log (X_{qnt-1}) \quad (3.22)$$

Deriving Wagner's law from the Cobb-Douglas production function

The Cobb-Douglas production function representing government spending on education (Y_{gt}) as a function of (a) government investment in higher education (X_{ht}) and (b) government spending on education (Y_{gt}) excluding government spending on higher education (X_{ht}) can be represented as follows:

$$Y_{gt} - X_{ht}^{\alpha^*} X_{hnt}^{\alpha^*} \quad (3.23)$$

Differentiation of equation (33) partially with respect to (wrt) government spending on higher education shows that the marginal physical product of higher education is a function of the average product of higher education and can be represented as follows:

$$\frac{\partial Y_{gt}}{\partial X_{ht}} = \alpha^* \frac{Y_{gt}}{X_{ht}} \quad (3.24)$$

Taking total differentiation of equation (33) provides:

$$\frac{dY_{gt}}{Y_{gt}} = \frac{\partial Y_{gt}}{\partial X_{ht}} \frac{X_{ht}}{Y_{gt}} \frac{dX_{ht}}{X_{ht}} + \frac{\partial Y_{gt}}{\partial X_{ht}} \frac{X_{hot}}{Y_{gt}} \frac{dX_{hnt}}{X_{hnt}} \quad (3.25)$$

Or

$$1 = \frac{\partial Y_{gt}}{\partial X_{ht}} \frac{dX_{ht}}{dY_{gt}} \left(\frac{Y_{gt}}{X_{hnt}}\right) \frac{X_{ht}}{Y_{gt}} + \frac{\partial Y_{gt}}{\partial X_{ht}} \frac{dX_{hot}}{dY_{gt}} \left(\frac{Y_{gt}}{X_{hot}}\right) \frac{X_{hot}}{Y_{gt}} \quad (3.26)$$

Comparing equations (34) and (36) implies that the marginal physical product of higher education is equal to the average physical product of higher education and can be represented as follows:

$$\frac{\partial Y_{gt}}{\partial X_{ht}} = 1. \frac{Y_{gt}}{X_{ht}} \quad (3.27)$$

Or

$$dlog (Y_{gt}) = 1. dlog (X_{ht}) \quad (3.28)$$

Since it follows that growth in government spending on education equals growth in government spending in higher education. Therefore, from equation (38) it implies that

$$Y_{gt} = \left(\frac{Y_{gt}}{X_{ht}}\right) X_{ht} \quad (3.29)$$

Hence, government funding of education can be represented as a function of government investment in higher education. Therefore, equation (39) can be rewritten as:

$$1 = \alpha_q \frac{X_{ht}}{Y_{gt}} \quad (3.40)$$

Similarly, an increase in government expenditure leads to an increase in output (Y) as follows:

$$1 = \alpha\ (G_t\ /\ Y_t) \tag{3.41}$$

where is a parameter that represents the effect of government investment in higher education on government funding of education.

Therefore, equation (41) implies that equation (1) can be rewritten as follows:

$$Y_t = G_t^{\alpha}\ (Y_t - G_t)^{\beta} \tag{3.42}$$

But government investment in higher education can be represented as a function of government funding of education. Therefore, equation (42) can be rewritten as follows:

$$log\ (G_t) = \frac{1}{\alpha}\ \log\ (Y_{gt}) - \frac{\beta}{\alpha}\ \log\ (Y_t - G_t) \tag{3.43}$$

Hence, the effect of economic growth on growth in government spending is usually greater than one.

Data types and data sources

The study makes use of secondary data over the 2005 to 2020 period on government spending in education sectors, collected from the MoES Strategic Plan and from the MoES Annual Performance Report for the periods (a) 2004 to 2015, (b) and (c) 2017/18 to 2019/20 (MoES, 2017; MoES, 2020). A dataset collected from the United Nations (2020) on real and nominal GPD was used in the computation of the implicit GDP deflator and also to deflate the respective variables in order to convert the available data into quantities of real variables as follows: government expenditures on (i) education (Y_{gt}), (ii) primary education (X_{pt}), (iii) secondary education (X_{st}), (iv) technical vocational education and training (X_{vt}), (v) higher (tertiary) education (X_{ht}) and (vi) administering education (X_{qt}).

Variables generated out of the data collected were government spending on education excluding (a) primary education (X_{pnt}), (b) secondary education (X_{snt}). (c) vocational education (X_{vnt}), (d) higher (tertiary) education (X_{hnt}) and (e) administering education (X_{qnt}).

Results

In conducting the regression analyses, the paper used the generalised least squares (GLS) technique. Tests on goodness of fit, serial correlation, joint effects, heteroscedasticity and significance of parameters were conducted by using the coefficient of determination (R^2), the Durbin Watson (DW) statistic, Fisher's (F) statistic, H statistic and t statistic, respectively. Meanwhile, is a vector that is used in multiplying through the regression equations in order to get rid of serial correlation and heteroscedasticity. After regressions and relevant statistical tests, all regression models obtained were found to be reliable for drawing conclusions and the relevant results are hereby reported as follows:

Effect of government spending on primary education investment

Equation (4.1) shows that a 1% increase in government spending on primary education investment could have caused public spending on education to rise by 0.35% in the short run. Test of equation 3.8.

$$\begin{array}{lll} 1 = 0.354(Y_{gt}/X_{pt-1}) & - 0.016(Y_{gt}/dY_{gt-1}). & (4.1) \\ t \qquad 42.92 & \qquad -6.98 & \end{array}$$

$$R^2 = 1, DW = 1.96, F = 6121, P = 2007 - 2020, N = 14, V = 1/\mathrm{d}(\mathrm{d}(Y_t^2)), H = 0.16$$

Effect of government spending on secondary education investment

Equation (4.2) shows that a 1% increase in government spending on secondary education investment could have caused public spending on education to rise by 0.086% in the short run. Test of equation 3.9.

$$\begin{array}{lll} 1 = 0.086(Y_{gt}/X_{st-1}) & - 0.032(Y_{gt}/dY_{gt-1}). & (4.2) \\ t \qquad 20.14 & \qquad -5.54 & \end{array}$$

$$R^2 = 0.99, DW = 1.73, F = 1368, P = 2007 - 2020, N = 14,\ V = 1/\mathrm{d}(\mathrm{d}(Y_t^2)), H = 0.24$$

Effect of government funding on technical and vocational education investment

Equation (4.3) shows that a 1% increase in government spending on technical and vocational education investment could have caused public spending on education to rise by 0.044% in the short run. It provides from test of equation 3.10.

$$\underset{t}{1} = \underset{20.06}{0.044(Y_{gt}/X_{vt-1})} - \underset{-5.98}{0.035(Y_{gt}/dY_{gt-1})}. \quad (4.3)$$

$$R^2 = 0.99, DW = 1.97, F = 1359, P = 2007 - 2020, N = 14,\ V = 1/\mathrm{d}(\mathrm{d}(Y_t^2)), H = 0.22$$

Effect of government funding on higher education investment

Equation (4.4) shows that a 1% increase in government spending on higher education investment could have caused public spending on education to rise by 0.063% in the short run. Test of equation 3.11.

$$\underset{t}{1} = \underset{30.90}{0.063(Y_{gt}/X_{ht-1})} - \underset{-4.52}{0.015(Y_{gt}/dY_{gt-1})}. \quad (4.4)$$

$$R^2 = 1, DW = 2.12, F = 3188, P = 2007 - 2020, N = 14,\ V = 1/\mathrm{d}(\mathrm{d}(Y_t^2)), H = 0.11$$

Effect of government spending on administration of education investment

Equation (4.5) shows that a 1% increase in government spending on education administration could have caused public spending on education to rise by 0.546% in the short run. Test of equation 3.12.

$$\underset{t}{1} = \underset{20.29}{0.546(Y_{gt}/X_{at-1})} - \underset{-0.70}{0.004(Y_{gt}/dY_{gt-1})}. \quad (4.5)$$

$$R^2 = 1, DW = 1.99, F = 1733, P = 2007 - 2020, N = 14,\ V = 1/\mathrm{d}(\mathrm{d}((Y/X_{st})^2)), H = 0.53$$

Effects of government spending on primary education investment

Equation (4.6) shows that a 1% increase in the growth of public spending could have caused growth in government spending on primary education to rise by 2.9% in the long run. Test of equation 3.18.

$$\underset{t}{dlog(X_{pt})} = \underset{17.08}{2.9dlog(Y_{gt-1})} - \underset{-10.76}{1.9dlog(Y_{gt-1} - X_{pt-1})} + \underset{41.91}{1.0ddlog(X_{pt})}. \quad (4.6)$$

$$R^2 = 1, DW = 2, F = 11647, P = 2008 - 2020, N = 13, V = 1/\mathrm{d}(\mathrm{d}((Y_{gt-1}/X_{st-1})^2)), H = 0.36$$

Effect of government spending on secondary education investment

Equation (4.7) shows that a 1% increase in growth of government spending on education could have caused public spending on secondary education growth to rise by 11.55% in the long run. Test of equation 3.19.

$$\underset{t}{dlog(X_{st})} = \underset{20.22}{11.45dlog(Y_{gt-1})} - \underset{-18.99}{10.5dlog(Y_{gt-1} - X_{st-1})} + \underset{365.4}{1.0ddlog(X_{st})}. \quad (4.7)$$

$$R^2 = 1, DW = 2.08, F = 7.86\times10^5, P = 2008 - 2020, N = 13,$$
$$V = 1/\mathrm{d}(\mathrm{d}((d(d(Y_{gt-1}/X_{st-1})))^2)), \quad H = 0.00$$

Effect of government funding on technical and vocational education investment

Equation (4.8) shows that a 1% increase in the growth of government spending on education could have caused public spending on T&V education growth to rise by 21.1% in the long run. Test of equation 3.20.

$$\underset{t}{dlog(X_{vt})} = \underset{36.44}{21.1dlog(Y_{gt-1})} - \underset{-35.28}{20.5dlog(Y_{gt-1} - X_{vt-1})} + \underset{72.98}{1.00ddlog(X_{vt})}. \quad (4.8)$$

$$R^2 = 0.99, DW = 2.16, F = 34223,\ P = 2008 - 2020, N = 13,$$
$$V = 1/\mathrm{d}(\mathrm{d}((d(d(Y_{gt-1}/X_{st-1})))^2)), H = 1.00$$

Effect of government funding on higher education investment

Equation (4.9) shows that a 1% increase in the growth of government spending on education could have caused public spending on higher education growth to rise by 16.6% in the long run. Test of equation 3.21.

$$\underset{t}{dlog(X_{ht})} = \underset{33.84}{16.6dlog(Y_{gt})} - \underset{-34.72}{15.7dlog(Y_{gt} - X_{ht-1})} + \underset{21.50}{0.9ddlog(X_{ht})}. \quad (4.9)$$

$$R^2 = 0.99, DW = 2.16, F = 774,\ P = 2007 - 2020, N = 14,\ V = 1/\mathrm{d}(\mathrm{d}(X_{ht}^2)), H = 0.13$$

Effect of government spending on administration of education

Equation (4.10) shows that a 1% increase in growth of government spending on education could have caused public spending on EA growth to rise by 1.946% in the long run. Test of equation 3.22.

$$dlog(X_{at}) = \underset{t}{1.94}dlog(Y_{gt}) - \underset{-237.8}{0.94}dlog(Y_{gt} - X_{at-1}) + \underset{21010}{1.00}ddlog(X_{at}). \quad (4.10)$$

$$R^2 = 1, DW = 1.95, F = 6.42\times10^{10}, P = 2009 - 2020, N = 12,$$
$$V = 1/\mathrm{d}(\mathrm{d}((d(d(X_{st-1}/X_{at-1})))^2)), H = 0.00$$

Effect of government investment in primary education on government education funding

Equation (4.11) shows that a 1% increase in government spending on education could have caused public spending on primary education to rise by 2.944% in the long run. See equation (3.40).

$$1 = \underset{t}{2.944}(X_{pt}/Y_{gt-1}). \quad (4.11)$$

$$R^2 = 0.98, DW = 2.21, P = 2007 - 2020, N = 14, V = 1/\mathrm{d}(\mathrm{d}(Y_t^2)), H = 0.46$$

Effect of government investment in secondary education on government education funding

Equation (4.12) shows that a 1% increase in government spending on education could have caused public spending on secondary education to rise by 12.722% in the long run. See equation (3.40).

$$1 = \underset{t}{12.722}(X_{st}/Y_{gt-1}), \quad (4.12)$$

$$R^2 = 1.00, DW = 2.09, P = 2007 - 2020, N = 14, V = 1/\mathrm{d}(\mathrm{d}(X_{ht}^2)), H = 0.26$$

Effect of government investment in technical and vocational education government education (in Uganda from 2005 to 2020.

Equation (4.13) shows that a 1% increase in government investment in education could have caused public spending on technical and vocational education to rise by 22.606% in the long run.

$$1 = \underset{t}{22.606}(X_{vt}/Y_{gt-1}), \quad (4.13)$$

$$R^2 = 1.00, DW = 1.85, P = 2007 - 2020, N = 14, V = 1/\mathrm{d}(\mathrm{d}((Y_t/Y_{gt})^2)), H = 0.01$$

Effect of government investment in higher education on government education funding

Equation (4.14) shows that a 1% increase in government investment in education could have caused public spending on higher education to rise by 15.849% in the long run. The results under equation (4.14) are used in testing of hypotheses in Equation 3.40.

$$1 = \underset{t}{15.849}(X_{ht}/Y_{gt-1}). \quad (4.14)$$

$$R^2 = 0.99, DW = 1.82, P = 2009 - 2020, N = 12, V = 1/\mathrm{d}(\mathrm{d}((d(d(Y_{gt}/X_{st})))^2)), H = 0.21$$

Effect of government funding of administration on its investment in education

Equation (4.15) shows that a 1% increase in government investment in education could have caused public spending on education administration to rise by 2.111% in the long run. The results under Equation (4.15) are used in testing of hypotheses in Equation 3.40.

$$1 = \underset{t}{2.111}(X_{at}/Y_{gt-1}). \quad (4.15)$$

$$R^2 = 0.99, DW = 2.03, P = 2008 - 2020, N = 13, V = 1/\mathrm{d}(\mathrm{d}(Y_{t-1}^2)), H = 0.04$$

Effect of government investment in primary education on output (GDP)

Equation (4.16) shows that a 1% increase in government spending on primary education could have caused output (GDP) to rise by 62.458% in the long run. See Equation (3.41).

$$1 = \underset{t}{62.458}\underset{20.22}{(X_{pt}/Y_t)}. \quad (4.16)$$

$$R^2 = 0.96, DW = 2.18, P = 2007 - 2020, N = 14,\ V = 1/\mathrm{d}(\mathrm{d}(Y_t^2)), H = 0.39$$

Effect of government investment in secondary education on output (GDP)

Equation (4.17) shows that a 1% increase in government spending on secondary education could have caused output (GDP) to rise by 275.25% in the long run. See Equation (3.41).

$$1 = \underset{t}{275.25}\underset{25.98}{(X_{st}/Y_t)}, \quad (4.17)$$

$$R^2 = 0.988, DW = 2.18, P = 2007 - 2020, N = 14,\ V = 1/\mathrm{d}(\mathrm{d}(Y_t^2)), H = 0.75$$

Effect of government investment in technical and vocational education on government investment in education (Y_{gt}) in Uganda from 2005 to 2020.)

Equation (4.18) shows that a 1% increase in government spending on technical and vocational education could have caused output (GDP) to rise by 561.1% in the long run. See equation (3.41).

$$1 = \underset{t}{561.11}\underset{44.67}{(X_{vt}/Y_t)}, \quad (4.18)$$

$$R^2 = 0.99, DW = 1.74, P = 2008 - 2020, N = 13,\ V = 1/\mathrm{d}(\mathrm{d}(X_{st}^2)), H = 0.46$$

Effect of government investment in higher education on output (GDP)

Equation (4.19) shows that a 1% increase in government spending on higher education could have caused output (GDP) to rise by 395.25% in the long run. Test of equation (3.41).

$$1 = \underset{t}{395.25}\underset{25.57}{(X_{ht}/Y_t)}. \quad (4.19)$$

$$R^2 = 0.978, DW = 2.17, P = 2007 - 2020, N = 14,\ V = 1/\mathrm{d}(\mathrm{d}(Y_t^2)), H = 0.51$$

Effect of government investment in administration of higher education on GDP

Equation (4.20) shows that a 1% increase in government spending on education administration could have caused output (GDP) to rise by 38.73% in the long run. See equation (3.41).

$$1 = \underset{t}{38.727}\underset{22.09}{(X_{at}/Y_t)}. \quad (4.20)$$

$$R^2 = 0.97, DW = 2.21, P = 2007 - 2020, N = 14,\ V = 1/\mathrm{d}(\mathrm{d}(Y_t^2)), H = 0.47$$

Effect of government investment in output (GDP)

Equation (4.21) shows that a 1% increase in government spending could have caused output (GDP) to rise by 20.096% in the long run. See equation (3.41).

$$1 = \underset{t}{20.096}\underset{22.75}{(X_{gt}/Y_{t-1})}. \quad (4.21)$$

$$R^2 = 0.97, DW = 2.18, P = 2007 - 2020, N = 14,\ V = 1/\mathrm{d}(\mathrm{d}(Y_{t-1}^2)), H = 0.48$$

Discussion of Results

Effects of government funding on levels of investment in education

We make use of both Wagner's law and Keynesian theories to analyse the relationships among income, government spending on primary, secondary, technical and vocational, and higher education, and administering education. Hence, from Table 1, under equations (1) to (5), by Keynes hypothesis, it can

be discerned that in the short run an increase of 32%, 8%, 4%, 6% and 50%, respectively, would cause government expenditures on education to increase by 1%, respectively, in government investment in primary, secondary, technical and vocational, higher and administering education in Uganda during the 2005 to 2020 period, ceteris paribus.

Table 1: Effects of government funding on levels of investment in education

List of Variables	Equations: 1–5		List of Variables
	Short Run		Government investment on:
	Actual	%	
Government Expenditure on education	0.354	32	Primary Education
	0.086	8	Secondary Education
	0.044	4	Tech. & Voc. Education
	0.063	6	Higher Education
	0.546	50	Education Administration
TOTAL	**1.093**	**100**	**TOTAL**

The results show that for two decades, development agencies have put more emphasis on primary and secondary education and, as a result, they have ignored the importance of tertiary education in promoting economic growth and poverty reduction. As mentioned above, our finding is in line with the Dakar summit on "Education for All" in 2000 which, for example, advocated for primary education only as a driver of broad social welfare which left tertiary education in the background (Bloom et al., 2014). However, this finding appears to operate in the short run.

Meanwhile, we postulate, in line with Keynesian theory, that an increase in government investment on primary, secondary, technical and vocational, and higher education, as well as administering education individually causes an increase in government spending on education (i.e. government income spent on education), as shown in Table 1 below.

Effects of government funding on levels of investment in education

Whereas, from Table 2, under equations (6) to (10), it can be discerned that in the long run an increase of 100% in government expenditures on education could have been caused by 6%, 20%, 39%, 31% and 4% increase, respectively, in government expenditures on primary, secondary, technical and vocational, and higher education, and administering education in Uganda during the 2005 to 2020 period, ceteris paribus.

In contrast to this early view, more recent evidence suggests that higher education is a determinant as well as a result of income, and can produce public and private benefits. Higher education may create greater tax revenue, increase savings and investment, and lead to a more entrepreneurial and civic society.

Table 2. Effects of government funding on levels of investment in education

List of Variables	Equations: 6 – 10		List of Variables
	Long Run		Government
	Actual	%	Investment on:
Government Expenditures on Education	3	6	Primary Education
	11	20	Secondary Education
	21	39	Tech. & Voc. Education
	17	31	Higher Education
Education	2	4	Education Administration
TOTAL	**54**	**100**	**TOTAL**

Contribution of government investment in education to government education funding

Meanwhile, from Table 3, under Wagner's law, we can conclude that in the long run an increase of 5%, 23%, 40%, 28% and 4%, respectively, in government investment in primary, secondary, technical and vocational, and higher education, as well as administering education could have been caused a 1%

increase in government expenditures on education during the given period, ceteris paribus. This implies that government policy should include paying special attention to providing adequate support to higher education as well as technical and vocational education.

Table 3. Contribution of government investment in education to government education funding.

Dependent Variables	Equations: 11 – 15		Independent Variables
Government	Long Run		Government
Expenditures on:	Actual	%	Investment in:
Education (Eq.11)	3	5	Primary Education
Education (Eq. 12)	13	23	Secondary Education
Education (Eq. 13)	23	40	Tech. & Voc. Education
Education (Eq. 14)	16	28	Higher Education
Education (Eq. 15)	2	4	Education Administration
TOTAL	**57**	**100**	**TOTAL**

Effects of government investment in levels of education on output (GDP)

In this section, our focus is mainly on higher education, which is an important form of investment in human capital. It can be regarded as a high level or a specialised form of human capital, the contribution of which to economic growth is very significant (Tilak, 2003, p.152). Also, tertiary education is considered a necessary and sufficient condition for the effective creation, dissemination and application of knowledge for building technical and professional capacity (Taylor, 2008, p.89). Indeed, new knowledge is a major source of competitive advantage, and it has been adopted as the most powerful driver of social and economic progress. Mueller (2006) adds that "knowledge is recognised as a crucial element of economic growth in addition to physical capital and labour", given that it can be commercially transformed into products and processes (i.e. value-added).

Results (Table 4) confirm that in the long run a 1% increase in Uganda's government investment in primary, secondary, technical and vocational, and higher education, as well as and administering education could have caused an increase of 5%, 21%, 42%, 30% and 3%, respectively, in GDP during the given period, ceteris paribus. These variables could have had an influence on GDP through the government multiplier or the respective education multipliers. This implies that government policy must include paying special attention to providing adequate support to higher education as well as technical and vocational education.

From Equation (5.21), we can conclude that in the long run an increase of 20.1% in GDP could have been caused by a 1% increase in government investment in education during the given period, ceteris paribus. Moreover, during the given period, government spending on education as a percentage of GDP for Uganda was on average 2%, while the world average was 4% (World Bank, 2022). This implies that low levels of government funding are responsible for the low levels of government investment in higher education in Uganda. This indicates that government must pay special attention to providing adequate support to the education sector in Uganda.

Table 4. Effect of government Investment in education on output (GDP)

Effect Variable	Equations:16-20		Causal Variable
	Long Run		
Gross Domestic Product (GDP)	Actual	%	Government Investment in:
GDP (Eq, 16)	62	5	Primary Education
GDP (Eq. 17)	275	21	Secondary Education
GDP (Eq. 18)	561	42	Tech. & Voc. Education
GDP (Eq. 19)	398	30	Higher Education
GDP (Eq. 20)	39	3	Education Administration
TOTAL	**1213**	**100**	**TOTAL**

Effects of government expenditures on economic growth

Empirical results portrayed by equation (21) shows that a 1% increase in government expenditures on education could have caused a 21.1% increase in economic growth in Uganda during the period 2007-2020. This finding is supported by Keynesian theory, which states that government expenditure plays a significant role in boosting economic growth. As a result, in the long run, if public expenditures are utilised well, they can assist in driving economic growth; this is because increased government spending increases GDP growth. Similarly, the result is in line with Shikomba et al.'s (2021) argument that government expenditure is an important determinant of economic growth. Hence, government policies that try to spend money more efficiently on productive sectors lead to economic growth. Governments are responsible for redistributing scarce resources through the production of goods, the purchase of commodities, and the provision of services (Ortiz-Ospina & Roser, 2016). Also, government expenditures are required to improve a country's population's knowledge, skills and education. Investing in human capital benefits the government because it increases labour force productivity, which leads to GDP growth (Shafuda & De, 2020).

The empirical results provide an answer to questions raised by some economists on whether government expenditure can improve the economy or not. Their questions arise from the scarcity of empirical evidence demonstrating the favourable effects of increased government expenditure on economic growth in developing nations (Nhlangwini & Tleane, 2019). In addition, our empirical finding is supported by Muktdair-Al-Mukit's (2012) study on the long-run relationship between public expenditure on education and economic growth in Bangladesh. He used an econometric model and time series data from 1995 to 2009. His findings indicate that public spending in education has a positive and significant impact on economic growth in the long run. Furthermore, he observed that a 1% increase in public expenditure in education contributes to a 0.34% increase in GDP per capita in the long run. This indicates that in the long run, government educational expenditure, through its impact on human capital, significantly and positively influences economic growth (Amaghionyeodiwe, 2018).

Summary

The two equations (A.1) and (A.2) in Appendix 1 show that in Uganda, an equilibrium could have existed during the 1972 to 2020 period because, in the long run, (a) increase in government spending could have caused output (GDP) to rise by 11% according to Keynesian hypothesis, while (b) increase in economic growth could have caused growth in public investment to rise by 11% according to Wagner law. These two equations justify equations (4.1) to (4.20) to be in agreement with the Keynesian hypothesis and Wagner's law, accordingly.

The paper corroborates the results with previous findings. Wagner postulates government expenditure as an endogenous factor caused by economic growth (Ansari et al., 1997; Ghazy et al., 2021). Appendix (A2) supports our law that economic growth would increase at a rate higher than the rate of the government sector. Whereas the results in Appendix (A1) support Wagner's law that in the real world, economic growth would increase at a rate higher than that of the government sector. According to Wagner's law, economic growth causes growth in the public sector. Wagner specifically takes spending on education and culture as part of government expenditure. He illustrates that government would do a more efficient job managing these entities than private enterprises, because they would be too large for private enterprises to undertake, since they require huge capital investments (Barro, 1988; Grier, 1989).

In addition, from Appendix 1 (A2) in equation (A2), it can be discerned that that a 1% increase in economic growth could have caused growth in public investment to rise by 11.130% in the long run in Uganda during the given period, ceteris paribus. The finding supports another major view offered by Keynes (1936), that government expenditure is an exogenous factor that can be manipulated by the government to stimulate the economy (Magazzino et al., 2015) as well as the level of national income (Ansari et al., 1997; Ghazy et al., 2021). Keynesian theory suggests that causality runs from public expenditure to economic growth.

The implication of this is that the Keynesian and Wagner theories are fundamentally different owing to the directions of the causal relationship between public expenditure and economic growth

(Kesavarajah, 2012). Meanwhile, Appendix 1 (A2) in equation (A2) shows that a 1% increase in economic growth could have caused growth in public investment to rise by 11.130% in the long run in Uganda during the given period, ceteris paribus.

This finding lends support to the voluminous literature since the seminal work of Wagner (1883) was first published. However, the relationship between economic growth and government spending still continues to be an unsettled issue in public economics (Wijeweera & Garis, 2009). Moreover, Wagner's law asserts that there are inherent tendencies for the activities of different layers of government to increase both intensively and extensively.

The theory assumes that in an economy, the government sector increases faster than output (GDP). However, graphical analysis with EViews supports our law. Hence, our law wins and shows that output increases faster than the government sector. Wagner's law emphasises long-term forces rather than short-term changes in public expenditure (Edame, 2014).

Hence, in the first instance, equation (4.20) supports the Keynesian hypothesis and shows that a 1% increase in government spending could have caused output (GDP) to rise by 20.096% in the long run in the country over the 2007 to 2020 period. The government spending multiplier shows a percentage change in GDP brought about by an increase in government expenditure (Adams & Wong, 2018). In Keynesian economics the multiplier effect of public expenditures on different economic activities of the national economic sectors (e.g. education) enables government to achieve the desired effects such as stability, growth and stimulation (Bista, 2016; Bista, 2021). Secondly, through the government multiplier, nearly all economies in the world, including Uganda, have a higher rate of public expenditure to GDP ratio (Bista & Sankhi, 2022).

Thirdly, through the respective multiplier effects of primary, secondary, tertiary and vocational training as well as higher education, the parameters in Table 1, under equations (1) to (5), could have been generated. Hence, by the Keynes hypothesis it can be discerned that in the short run an increase of 32%, 8%, 4%, 6% and 50%, respectively, in government expenditures on education could have been caused by a 1% increase, respectively, in government investment in primary, secondary, technical and vocational, and higher education, as well as administering education.

Fourthly, from Table 1, under equations (6) to (10), it can be discerned that in the long run, through the government multiplier of spending at different educational levels, an increase of 100% in government expenditures on education could have been caused by a 6%, 20%, 39%, 31% and 4% increase, respectively, in government expenditures on primary, secondary, technical and vocational, and higher education, as well as administering education in Uganda during the 2005 to 2020 period, ceteris paribus.

Lastly, from Table 2 in equations (11) to (15) under Wagner's law, we confirm, through the multiplier effects at the five levels of education, that in the long run a 1% increase in Uganda's government investment in primary, secondary, technical and vocational, and higher education, as well as administering education could have caused an increase of 5%, 13%, 23%, 16% and 2%, respectively, in GDP during the given period, ceteris paribus.

Conclusion

By using the Cobb-Douglas production function, the paper derives models for both the Keynesian theory and Wagner's law and uses them to test relationships between government funding of education and government investment in higher education in Uganda during the 2005 to 2020 period. Empirical findings show that both the Keynesian and Wagner regression models provide similar parameter results, implying the existence of equilibrium in the economy. Meanwhile, Table 2 shows results of long-run issues under Wagner's law. From Table 2, we can deduce that an increase of 5%, 21%, 42%, 30% and 3%, respectively, in government investment in primary, secondary, technical and vocational, as well as higher and administering education could have been caused by a 1% increase in GDP during the given period, ceteris paribus.

Similarly, Table 1, under equations (4.6) to (4.10), by Keynes' hypothesis, it can be discerned that in the short run an increase of 6%, 20%, 39%, 31% and 4%, respectively, in government expenditures on education could have been brought about by a 1% increase, respectively, in government investment in primary,

secondary, technical and vocational, as well as higher, besides administering education in Uganda during the 2005 to 2020 period, ceteris paribus. These results also show the existence of equilibrium conditions in Uganda between Wagner's law and Keynes' hypothesis. However, we find that national output (GDP) rises faster than public expenditure.

Hence, government policy in Uganda needs to pay special attention (a) to providing adequate support to the education sector, particularly to higher education as well as technical and vocational education; (b) to providing adequate funding to the education sector in Uganda; (c) to manipulating its spending while aiming at stimulating the economy; and (d) to making drastic improvements in the education infrastructure (accommodation facilities for staff, classrooms, ICT facilities, provision of computers to individual students, uplifting the status of libraries and improvement of social networks) at all levels of education.

We have some few limitations of the study. One, the study is limited to Uganda. Two, in our empirical analysis, the study covers the 2005 to 2020 period. Three, the study is based on time series analysis and employs the GLS method in data analysis. Four, the study is based on Wagner's law and Keynesian theory.

However, some relevant findings in our study support the theory we have advanced that an "increase in national output is higher government expenditure". Lastly, our study focuses on the relationship between government funding of education and government investment in higher education in Uganda. The study also examines the relationship between government spending and economic growth because government spending is composed of government expenditures on primary, secondary, technical and tertiary education as well as the administration of education. Hence, our study concludes that government expenditure increases national income through a multiplier effect on aggregate demand, leading to economic growth.

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Appendix 1. Tests of Wagner Law and Keynesian Hypothesis

A1. Effect of Government Funding on Economic Growth in Uganda: Keynes Hypothesis

Equation (A.1) shows that a 1% increase in government spending could have caused output (GDP) to rise by 11.008% in the long run in Uganda during the 1972 to 2020 period, ceteris paribus. See Equation (3.41) for verification (validation).

$$\underset{t}{1} = \underset{5144}{11.008}(G_t/Y_t). \qquad \text{(A1)}$$

$$R^2 = 1.0000, DW = 1.847, Period = 1972 - 2020, N = 47,\ V = 1/\mathrm{d}(\mathrm{d}(Y_{t-1}^2)), H = 0.005$$

A2. Effect of Economic Growth on Public Investment in Uganda: Wagner Law

Equation (A2) shows that a 1% increase in economic growth could have caused growth in public investment to rise by 11.130% in the long run in Uganda during the given period, ceteris paribus.

$$\underset{t}{dlog(G_t)} = \underset{255.30}{11.130}dlog(Y_t) - \underset{-234.87}{10.128}dlog(Y_t - G_t). \qquad \text{(A2)}$$

$$R^2 = 1.0000, DW = 2.18, Period = 1972 - 2020, N = 47,\ V = 1/\mathrm{d}(\mathrm{d}(Y^2)), H = 0.019$$

Appendix 2. Government Sector Increases at a Rate Higher than that of Output

Our contribution is to give theoretical and empirical answers to one critical question asking whether Wagner's law would cause the government sector to grow at a rate higher than the rate of the economic growth. To provide meaningful answers we consider three variables: output denoted as gross domestic product GDP), government spending (Y) and output excluding government spending (Q), where $Q = Y - G = Q\,[Y\,(G)]$, Thus the model can be represented as follows:

$$Q[Y(G)] + G = Y(G). \qquad \text{(A3)}$$

Equation (A3) shows that the expression $Y(G)$ stress that solution Y depends on G. By using chain rule, differentiation of Equation (A3) with respect to G provides

$$\frac{\partial Q}{\partial Y}\frac{\partial Y}{\partial G} + 1 = \frac{\partial Y}{\partial G}. \qquad \text{(A4)}$$

Rearranging Equation (A4) give rise to an expression for government multiplier as follows:

$$\left(1-\frac{\partial Q}{\partial Y}\right)\frac{\partial Y}{\partial G}=1. \quad \text{(A5)}$$

Hence, the government multiplier can be represented as follows:

$$\partial Y=\frac{1}{\left(1-\frac{\partial Q}{\partial Y}\right)}\partial G=\frac{1}{\left(\frac{\partial G}{\partial Y}\right)}\partial G=\frac{1}{(g)}\partial G=\frac{1}{g}\partial G. \quad \text{(A6)}$$

Taking the integral of Equation (A6) the government multiplier can be rewritten as follows:

$$Y=\frac{1}{g}G. \quad \text{(A7)}$$

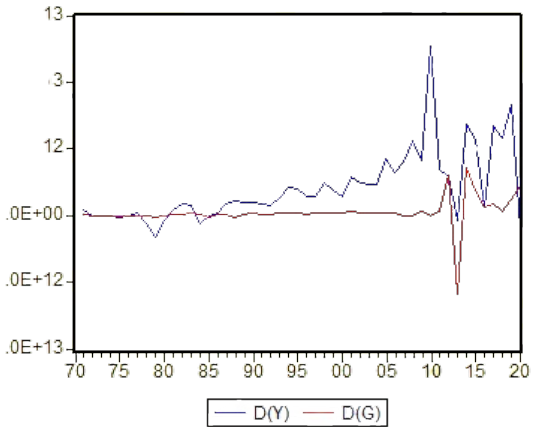
Implying that increase in government will always cause output to increase more than output would cause government expenditure to grow as given by

$$1=\left(\frac{1}{g}\right)\frac{G}{Y}. \quad \text{(A8)}$$

Hence, Equation (A8) proves Keynesian hypothesis to be untrue since it shows that government sector increases at a rate higher than the rate at which output increases, since $g<1$ while $(1/g<1$. Therefore, according to our empirical finding we reject Wagner's Law that "public expenditure rises faster than national output (GDP)" (Gatsi, et al., 2019).

However, Wagner's Law and Keynesian Hypothesis are two sides of the same coin (See Equation (2.5) and (2.6) under review of literature). Thus, government sector does not increase at a higher output. Instead, it is increase in national output that is higher than increase in government sector. Therefore, in our graphical analysis involving comparison of movements in $d(G)$ and $d(Y)$ we find that economic growth increases at a higher rate than the rate of government sector. Thus, (See Figure 1) shows, our law passes the graphical test. But Wagner's Law fails the graphical test.

Figure 1. Increase in National Output is Higher than Increase in Government Expenditure



Consequently, we test Wagner's Law which postulates "public expenditure rises faster than national output" by regressing $(\partial G)/(\partial Y)$ on 1 and by regressing $(\partial Y)/(\partial G)$ on 1, using GLS technique. Our empirical findings depicted by results in Equations (A9) and (A10) show that $(\partial G)/(\partial Y) = 0.0897$ and $(\partial Y)/(\partial G) = 11.118$. Therefore, we refute Wagner's Law that "public expenditure rises faster than national output." Hence, we accept our law that national output (GDP) rises faster than public expenditure (See Equations (A9) and (A10) for verification.).

$$\frac{\partial G}{\partial Y}=0.0897. \quad \text{(A9)}$$
$$t \qquad 143$$
$$R^2=0.9978, DW=1.82, Period=1975-2020, N=45,$$
$$V=1/\mathrm{d}(\mathrm{d}((d(Y/C_n))^2)), H=0.033$$

$$\frac{\partial Y}{\partial G}=11.118. \quad \text{(A10)}$$
$$t \qquad 98.85$$
$$R^2=0.9951, DW=1.87, Period=1973-2020, N=48,\ V=1/\mathrm{d}(\mathrm{d}(Y^2_{d(-1)})), H=0.066$$